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COMMENT

Isotropic chiral media and scalar Hertz potentials

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Abstract. A new approach to electromagnetic field representations in isotropic chiral media is presented. It is shown that by a simple transformation of the fields the constitutive relations of the isotropic chiral medium (which include differential operators) can be transformed into those of a special form of bianisotropic media. It is finally found that the electromagnetic field can be represented by two scalar Hertz potentials which are solutions of a system of second-order differential equations. The Green functions of this system are calculated.

In a recent paper by Varadan *et al* (1987) the electromagnetic fields in a special class of media with constitutive relations of the form

$$\boldsymbol{D} = \varepsilon(\boldsymbol{E} + \beta\nabla\times\boldsymbol{E}) \tag{1a}$$

$$\boldsymbol{B} = \mu(\boldsymbol{H} + \beta\nabla\times\boldsymbol{H}) \tag{1b}$$

are investigated. These so-called isotropic chiral media (β measures the chirality) are of certain interest in physical and stereochemistry regarding the behaviour of electromagnetic waves propagating through optically active substances. For more details about the physical and chemical background of the discussed media one is referred to Varadan *et al* (1987) and references cited therein. In this comment we will focus on some general comments about the constitutive relations and on the representation of the electromagnetic field.

The appearance of a differential operator in the constitutive relations ($1a$, b) makes the media spatially dispersive by definition, i.e. $\boldsymbol{D}$ at a point $\boldsymbol{x}$ depends on the electric field $\boldsymbol{E}$ at other points $\boldsymbol{x}'$ (and the same applies to $\boldsymbol{B}$). Although some results are known for spatially dispersive media (see, for example, Felsen and Marcuvitz (1973)) isotropic chiral media have not been thoroughly investigated in the past. However it is easy to see that the introduction of new electromagnetic field vectors $\boldsymbol{E}'$ and $\boldsymbol{H}'$ by virtue of the transformation

$$\boldsymbol{E}' = \boldsymbol{E} - \mathrm{i}\omega\mu\beta\boldsymbol{H} \tag{2a}$$

$$\boldsymbol{H}' = \boldsymbol{H} + \mathrm{i}\omega\varepsilon\beta\boldsymbol{E} \tag{2b}$$

(the harmonic time dependence is $\exp(-\mathrm{i}\omega t)$, ω being the frequency) leads to the following form of Maxwell's equations for the primed fields:

$$\nabla\times\boldsymbol{H}' + \mathrm{i}\omega\boldsymbol{D}' = \boldsymbol{J} \tag{3a}$$

$$\nabla\times\boldsymbol{E}' - \mathrm{i}\omega\boldsymbol{B}' = 0 \tag{3b}$$

whereby the constitutive relations take the form

$$\boldsymbol{D}' = \varepsilon\lambda(\boldsymbol{E}' + \mathrm{i}\omega\mu\beta\boldsymbol{H}') \tag{4a}$$

$$\boldsymbol{B}' = \mu\lambda(\boldsymbol{H}' - \mathrm{i}\omega\varepsilon\beta\boldsymbol{E}') \tag{4b}$$

with $\lambda = 1/(1-k^2\beta^2)$ and $k^2 = \omega^2\varepsilon\mu$.

Maxwell's equations (3*a*, *b*) and the constitutive relations (4*a*, *b*) for the primed fields are then equivalent to the original Maxwell's equations (which have the same form as (3*a*, *b*) with the primes omitted) and their respective constitutive relations (1*a*, *b*). This holds true if the primed and the unprimed fields are linked by the transformation (2*a*, *b*). But the new constitutive relations (4*a*, *b*) are now easily recognised as those of a special form of general spatially non-dispersive bianisotropic media

$$\boldsymbol{D}' = \boldsymbol{\varepsilon}\cdot\boldsymbol{E}' + \boldsymbol{\xi}\cdot\boldsymbol{H}' \tag{5a}$$

$$\boldsymbol{B}' = \boldsymbol{\zeta}\cdot\boldsymbol{E}' + \boldsymbol{\mu}\cdot\boldsymbol{H}' \tag{5b}$$

which have been treated extensively in the literature (Felsen and Marcuvitz 1973, Suchy *et al* 1985, Weiglhofer 1987).

It has recently been shown (Weiglhofer 1987) that Maxwell's equations in general bianisotropic media can be represented by two scalar Hertz potentials (which is a familiar result for homogeneous isotropic media) provided that the constitutive tensors $\boldsymbol{\varepsilon}$, $\boldsymbol{\mu}$, $\boldsymbol{\xi}$ and $\boldsymbol{\zeta}$ have the structure

$$\mathbf{a} = a_1\mathbf{I} + a_2(\boldsymbol{b}\times\mathbf{I}) + a_3\boldsymbol{b}\boldsymbol{b} \tag{6}$$

where $\mathbf{I}$ is the unit tensor and $\boldsymbol{b}$ is an arbitrary unit vector. This result holds true even for inhomogeneous media where the parameters a_1, a_2 and a_3 are functions of the coordinate in the direction of $\boldsymbol{b}$.

For a detailed mathematical treatment of isotropic chiral media with the scalar Hertz potential technique we refer to Weiglhofer (1988). Here the results are given which have been obtained by solving Maxwell's equations for the primed fields $\boldsymbol{E}'$ and $\boldsymbol{H}'$ and then transforming back to $\boldsymbol{E}$ and $\boldsymbol{H}$. Confining ourselves to an electric source density

$$\boldsymbol{J}(\boldsymbol{x}) = J(\boldsymbol{x})\boldsymbol{b} \tag{7}$$

the representation of the electromagnetic field is given by

$$\boldsymbol{E} = \nabla\times\nabla\times u\boldsymbol{b} + k^2\beta\lambda(\nabla\times u\boldsymbol{b}) + \mathrm{i}\omega\mu\lambda(\nabla\times v\boldsymbol{b}) + \boldsymbol{J}/\mathrm{i}\omega\varepsilon \tag{8a}$$

$$\boldsymbol{H} = \nabla\times\nabla\times v\boldsymbol{b} + k^2\beta\lambda(\nabla\times v\boldsymbol{b}) - \mathrm{i}\omega\varepsilon\lambda(\nabla\times u\boldsymbol{b}) \tag{8b}$$

where the scalar Hertz potentials u and v must be solutions of the system of partial differential equations

$$(\nabla^2 + k'^2)u + 2\mathrm{i}\omega\mu\beta\lambda^2k^2v = J/\mathrm{i}\omega\varepsilon \tag{9a}$$

$$-2\mathrm{i}\omega\varepsilon\beta\lambda^2k^2u + (\nabla^2 + k'^2)v = 0 \tag{9b}$$

with the Laplace operator ∇^2 and $k'^2 = k^2\lambda^2(1+\beta^2k^2)$.

The solution of (9*a*, *b*) can be found easily and one gets for the corresponding Green functions (i.e. J is replaced by $-\delta(\boldsymbol{x}-\boldsymbol{x}')$)

$$u = (1/8\pi\mathrm{i}\omega\varepsilon r)[\exp(\mathrm{i}\gamma_1 r) + \exp(\mathrm{i}\gamma_2 r)] \tag{10a}$$

$$v = (1/8\pi k r)[\exp(\mathrm{i}\gamma_1 r) - \exp(\mathrm{i}\gamma_2 r)] \tag{10b}$$

with $\gamma_1 = k/(1-\beta k)$, $\gamma_2 = k/(1+\beta k)$ and $r = |\boldsymbol{x}-\boldsymbol{x}'|$.

Thus we have been successful in deriving an electromagnetic field representation in terms of two scalar Hertz potentials. Moreover, this analysis achieves this end by directly solving Maxwell's equations and therefore it seems to be more advantageous than the method of Varadan *et al* (1987) who find the Green tensor representation (and therefore the electromagnetic field representation) by extracting differential operators from Fourier integrals, a step which does not seem rigorous from a strict mathematical point of view.

Finally one might add that the scalar Hertz potential technique is capable of treating stratified media, i.e. the constitutive parameters are allowed to depend on one spatial coordinate.

References

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